Journal of Engineering Mathematics 22: 253–265 (1988)

Optimal strength of a flexible high-pressure hose with two steel braids

C.M.J. HOGENBIRK, B.A. VAN DEN HORN, E.E.R. JAGER* and M. KUIPERS
*Department of Mathematics and Computing Science, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands (*at present with N.V. Nederlandse Gasunie, Groningen)*

Received 21 January 1988; accepted 5 February 1988

Abstract. The flexible high-pressure hose we consider in this paper, is a rubber tube reinforced by two steel braids. Each of the latter consists of two families of thin steel wires which are wound helicoidally around the cylinder. In order to achieve an optimal strength of the tube, the stresses in the wires of the two braids must be much the same. We investigate whether this can be accomplished by a proper choice of the braid angles of the two braids. Throughout this paper we apply a linear theory.

1. Introduction

High-pressure hoses which allow some bending, usually consist of a rubber tube the wall of which is reinforced by means of one or more steel braids. Thin rubber sheets separate these braids of which each is composed of thin steel wires. Frequently these wires are gathered up in two families of flat ribbons that are interwoven along helicoidal lines on the braid cylinders. The weaving pattern is regular and symmetric (Fig. 1). In this paper we shall consider this hose type. We define the pitch angle of a braid as the acute angle φ between a tangent to a helicoidal curve of a braid and a normal cross-section of the hose. This angle plays a significant role in what follows.

If we pull at a piece of an isolated braid we note that it gives way rather easily through a distance of, say, one-sixteenth of its length. It seems to behave as if it were a non-linear spring, which blocks up at a maximal extension. This flexibility stems from the concertina-like motion the braid ribbons are able to perform, virtually without any force. The latter develops as the motion progresses and arises from the constraint that one family of wires constitutes in relation to the motion of the other one, and vice versa. The pliancy of the braid, combined with the small stiffness of the rubber tube, accounts for the comparatively great flexibility of the hose, particularly in bending.

In addition to the demand of a sufficient flexibility, a tube must, of course, be strong enough. It has to withstand rather high pressures of, say, 300 bars. Obviously it is the steel reinforcement which carries this load. The contribution of the rubber to the strength is small. The inner rubber cylinder serves merely to prevent oil from leaking through the gaps between the steel wires. The outer rubber cylinder protects the tube against mechanical damage from external agencies. However, the steel wires can be loaded in tension exclusively. This means that a braid can cope with a specific loading system only. The tangential and axial tensile stresses have to bear to one another in a certain ratio following from the pitch angle φ of the braid. If this condition is not complied with, the braid will adapt itself more or less to the load by means of its shearing mechanism. The deformation stops when the new pitch

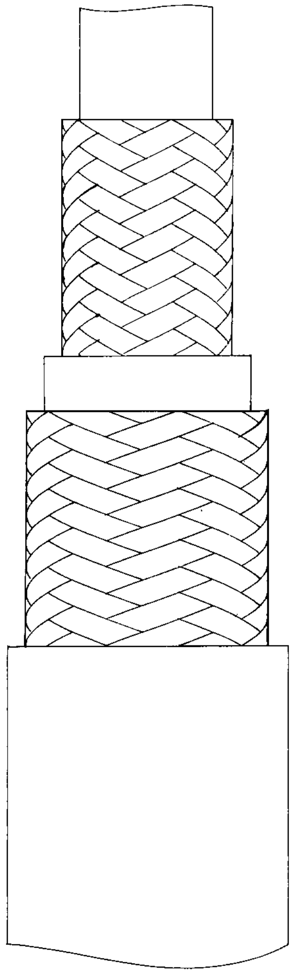

Fig. 1. Hose with two steel braids.

angle conforms to the external loading. It is clear that the rubber tubes are compelled to participate in the distortion. Hence, on these conditions it is to be expected that the rubber adds slightly to the overall stiffness of the hose, although its contribution to the strength can be neglected.

When the internal pressure increases, failure of the hose occurs at the so-called burst pressure. We mention three main causes giving rise to a break-down of a hose. First of all, the inner rubber tube may be blown through the gaps between the wires of the braids. Secondly, some wires of a braid rupture as a result of too high tensile stresses and, finally, fracture of braids and rubber occurs in the immediate neighbourhood of a connection nipple. The first phenomenon is usually prevented by making the inner rubber cylinder sufficiently thick and stiff. If one wants to save material by reducing the rubber thickness, then one has to pay attention to it. The connection between a hose and a nipple, mentioned as the third item, may be a source of trouble, for a nipple is forged onto a hose end and this process involves very large permanent deformations of the hose end. Only by means of a careful design of the nipple, aimed at a transition from the nipple to the hose as smooth as possible, it is feasible to avoid unacceptable concentration of stresses in the clamped part of the hose. Experience shows that bursting of a hose generally results from fracture of some braid wires. From this it follows that it is of primary importance to try for a uniform spreading of the load over the various braids when two or more of these are applied. To that end in this paper we consider a hose with two steel braids. We shall use the same assumptions as in Section 5 of [1], where we calculated the strains and stresses in a hose with only one braid. This means that we shall use the theory of thick-walled cylinders for the rubber tubes, whereas the braids are supposed to be infinitely thin. Furthermore we shall neglect also the mutual obstruction of the two families of each braid with respect to each others motion.

In our model the intermediate thin rubber sheet transfers a part of the load from the inner to the outer braid. Hence, it is clear that its thickness and compressibility will play an important role in this process. Since the thin sheet will tend to fill the voids between the wires of the braids, it is difficult to obtain a reliable estimate of its effective thickness and compressibility. In particular, the value of the Poisson's ratio we have chosen is disputable. However, we shall focus our attention mainly to the effect of variations of the two braid angles on the spreading of the load over the braids.

2. Mathematical analysis

We refer our analysis to a cylindrical coordinate system r, ϑ, z (Fig. 2) and consider a tube consisting of three rubber cylinders. They occupy the regions $r_1 < r < r_2$, $r_2 < r < r_3$ and $r_3 < r < r_4$, respectively. The length of the cylinders does not occur in our calculations explicitly. The three rubber cylinders are separated by two steel braids at $r = r_2$ and $r = r_3$. Each steel braid consists of two families of wires, as described in the introduction. The braid angles at $r = r_2$ and $r = r_3$ are φ_1 and φ_2, respectively.

In the sequel we apply a linear small-displacement theory and we assume the rubber of the three cylinders to be Hookean materials with Young's moduli $E^{(i)}$ and Poisson's ratio's $\nu^{(i)}$.

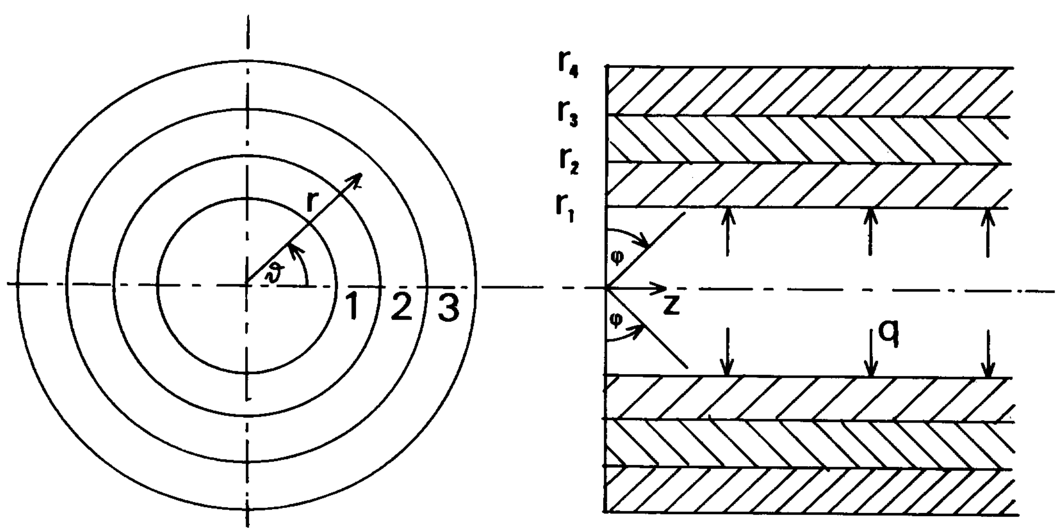

Fig. 2. Hose (inner radius r_1, outer radius r_4) with infinitely thin steel braids at $r = r_2$ and $r = r_3$, loaded by an internal pressure q. The braid angle is the pitch angle φ.

Here and in the following we use a superscript i (i = 1, 2, 3), to refer to the inner, middle and outer cylinder, respectively. The radial displacement is denoted by $u^{(i)}$ and the axial one by $w^{(i)}$. Neglecting any edge effects, we suppose $u^{(i)} = u^{(i)}(r)$ and $w^{(1)} = w^{(2)} = w^{(3)} = w(z)$, so that the axial strain is uniform. Then, according to the well-known theory of thick-walled cylinders, e.g. see [2], the normal stresses $t_{rr}^{(i)}$, $t_{\vartheta\vartheta}^{(i)}$ and $t_{zz}^{(i)}$ follow from

$$
\begin{aligned}
t_{rr}^{(i)} &= \lambda^{(i)}\left\{\frac{\mathrm{d}u^{(i)}}{\mathrm{d}r} + \frac{u^{(i)}}{r} + \frac{\mathrm{d}w}{\mathrm{d}z}\right\} + 2\mu^{(i)}\frac{\mathrm{d}u^{(i)}}{\mathrm{d}r},\\
t_{\vartheta\vartheta}^{(i)} &= \lambda^{(i)}\left\{\frac{\mathrm{d}u^{(i)}}{\mathrm{d}r} + \frac{u^{(i)}}{r} + \frac{\mathrm{d}w}{\mathrm{d}z}\right\} + 2\mu^{(i)}\frac{u^{(i)}}{r}, \qquad (2.1)\\
t_{zz}^{(i)} &= \lambda^{(i)}\left\{\frac{\mathrm{d}u^{(i)}}{\mathrm{d}r} + \frac{u^{(i)}}{r} + \frac{\mathrm{d}w}{\mathrm{d}z}\right\} + 2\mu^{(i)}\frac{\mathrm{d}w}{\mathrm{d}z}, \quad i = 1, 2, 3,
\end{aligned}
$$

where $\lambda^{(i)}$ and $\mu^{(i)}$ are the Lamé's parameters. They satisfy the equilibrium equations

$$
\frac{\mathrm{d}t_{rr}^{(i)}}{\mathrm{d}r} + \frac{t_{rr}^{(i)} - t_{\vartheta\vartheta}^{(i)}}{r} = 0, \quad \frac{\mathrm{d}t_{zz}^{(i)}}{\mathrm{d}z} = 0, \quad i = 1, 2, 3. \qquad (2.2)
$$

Upon substituting (2.1) into (2.2), the second equation is satisfied identically, while $(2.2)^1$ yields

$$
\frac{\mathrm{d}^2u^{(i)}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}u^{(i)}}{\mathrm{d}r} - \frac{1}{r^2}u^{(i)} = 0, \quad i = 1, 2, 3. \qquad (2.3)
$$

The solutions are

$$
u^{(i)} = A^{(i)}r + \frac{B^{(i)}}{r}, \quad i = 1, 2, 3, \qquad (2.4)
$$

where $A^{(i)}$ and $B^{(i)}$ are integration constants, which will be determined later on. In the sequel we will use Young's moduli $E^{(i)}$ and Poisson's ratio's $\nu^{(i)}$ instead of $\lambda^{(i)}$ and $\mu^{(i)}$ applying the relations

$$
\lambda^{(i)} = \frac{E^{(i)}\nu^{(i)}}{(1 + \nu^{(i)})(1 - 2\nu^{(i)})} \quad \text{and} \quad \mu^{(i)} = \frac{E^{(i)}}{2(1 + \nu^{(i)})}.
$$

Since the radial displacement is continuous at $r = r_2$ and $r = r_3$ we have

$$
u^{(1)}(r_2) = u^{(2)}(r_2), \quad u^{(2)}(r_3) = u^{(3)}(r_3). \qquad (2.5)
$$

Now turning to the braid, we first assume that the wires are inextensible. This means that, in a linear context,

$$
\frac{u^{(i)}(r_{i+1})}{r_{i+1}} + \frac{\mathrm{d}w}{\mathrm{d}z}\tan^2\varphi_i = 0, \quad i = 1, 2. \qquad (2.6)
$$

Furthermore, the internal forces N_1 and N_2 in the braids, defined as the force in each family measured per unit of length in a direction perpendicular to the wires in that family, cannot be derived from a Hooke's law. Hence, they constitute two unknown quantities in addition to the six constants $A^{(i)}$ and $B^{(i)}$ introduced above. Moreover, the uniform axial strain $\mathrm{d}w/\mathrm{d}z$ is not known beforehand. Altogether there are nine quantities to be determined from nine equations, viz. (2.5), (2.6) and the following conditions

$$t_{rr}^{(1)}(r_1) = -q, \tag{2.7}$$

$$r_{i+1}\{t_{rr}^{(i+1)}(r_{i+1}) - t_{rr}^{(i)}(r_{i+1})\} = 2N_i \cos^2 \varphi_i, \quad i = 1, 2, \tag{2.8}$$

$$t_{rr}^{(3)}(r_4) = 0, \tag{2.9}$$

$$q\pi r_1^2 = 2\pi \sum_{i=1}^{2} 2r_{i+1} N_i \sin^2 \varphi_i + 2\pi \sum_{i=1}^{3} \int_{r_i}^{r_{i+1}} t_{zz}^{(i)}(\zeta)\zeta \,\mathrm{d}\zeta. \tag{2.10}$$

The latter equation results from the assumption that the axial force exerted by the fluid in the tube, is carried completely by the rubber cylinders and the braids. Numerical results will be given in the following section.

If we drop the inextensibility of the steel wires and assume that they are elastic, the above analysis can be adjusted easily. The equation (2.6) has to be replaced by

$$e_i = \frac{\mathrm{d}w}{\mathrm{d}z} \sin^2 \varphi_i + \frac{u^{(i)}(r_{i+1})}{r_{i+1}} \cos^2 \varphi_i, \quad i = 1, 2, \tag{2.11}$$

where e_i is the strain in a wire. Subsequently we calculate the force N_i by means of Hooke's law, yielding

$$N_i = \frac{E_i^s d_i^2 n_i e_i}{16\, r_{i+1} \sin \varphi_i}, \quad i = 1, 2, \tag{2.12}$$

in which E_i^s = Young's modulus of the steel in braid i, d_i = the diameter of the wires in braid i, n_i = total number of wires in a cross-section of brain i (total means twice the number in one family).

The elimination of e_i from (2.11) and (2.12) yields an equation which replaces (2.6) in the above analysis. The other equations remain unchanged.

In the sequel we shall calculate the change $\Delta\varphi_i$ of the pitch angle φ_i, $i = 1, 2$. In the case of inextensible wires this quantity follows from

$$\Delta\varphi_i = \frac{\mathrm{d}w}{\mathrm{d}z} \tan \varphi_i, \quad i = 1, 2. \tag{2.13^1}$$

For elastic wires we have

$$\Delta\varphi_i = \sin \varphi_i \cos \varphi_i \left(-\frac{u^{(i)}(r_{i+1})}{r_{i+1}} + \frac{\mathrm{d}w}{\mathrm{d}z} \right). \tag{2.13^2}$$

Finally, in the sequel we are concerned with the force F_i in each wire of braid i. This quantity follows from

$$F_i = \frac{4\pi r_{i+1} N_i \sin \varphi_i}{n_i}, \quad i = 1, 2. \tag{2.14}$$

3. Numerical calculations

The numerical calculations we have performed refer to a specific hose: Trelleborg, type 320/10. Its main dimensions are

$$\begin{aligned} r_1 &= 4.75\,\text{mm}, \quad r_3 = 8.00\,\text{mm}, \quad d_1 = 0{,}3\,\text{mm}, \\ r_2 &= 7.00\,\text{mm}, \quad r_4 = 9.50\,\text{mm}, \quad d_2 = 0{,}3\,\text{mm}. \end{aligned} \tag{3.1}$$

The number of wires in a cross-section of the braids is

$$n_1 = 140 \quad \text{and} \quad n_2 = 160. \tag{3.2}$$

The maximum working pressure retained in the calculations, is $q = 33$ N/mm^2.

As to the elastic properties of the three rubber cylinders, we know that the value of the modulus of compression $\kappa^{(2)}$ of the intermediate rubber cylinder is of crucial importance. Since $\kappa^{(2)} = E^{(2)}/[3(1 - 2\nu^{(2)})]$ and $\nu^{(2)} \approx 1/2$, we see that $\nu^{(2)}$ plays a significant role.

Table 1. Trelleborg hose 320/10

Stresses, strains and displacements for the case:
$E^{(1)} = E^{(2)} = E^{(3)} = 4\,\text{N/mm}^2$, $E_1^s = E_2^s = \infty$,
$\nu^{(1)} = \nu^{(3)} = 0.4997$, $\nu^{(2)} = 0.4999$, $n_1 = 140$, $n_2 = 160$, $q = 33$ N/mm^2,
$\varphi_1 = 33.31°$, $\varphi_2 = 37.08°$.

i	r (mm)	$t_{rr}^{(i)}$ (N/mm^2)	$t_{\vartheta\vartheta}^{(i)}$ (N/mm^2)	$u^{(i)}$ (mm)	$e_{rr}^{(i)}$	$e_{\vartheta\vartheta}^{(i)}$
	4.750	−33.000 00	−32.913 0	0.037 95	−0.024 79	0.007 99
1	5.875	−32.985 00	−32.928 0	0.013 59	−0.019 11	0.002 31
	7.000	−32.980 00	−32.936 0	−0.005 98	−0.015 95	0.000 85
	7.000	−13.886 20	−13.880 0	−0.005 98	−0.003 21	−0.000 85
2	7.500	−13.885 80	−13.880 4	−0.007 54	−0.003 05	−0.001 00
	8.000	−13.885 50	−13.880 7	−0.009 04	−0.002 93	−0.001 13
	8.000	0.000 11	−0.000 64	−0.009 04	−0.000 85	−0.001 13
3	8.750	0.000 05	−0.000 58	−0.009 86	−0.000 87	−0.001 10
	9.500	0.000 00	0.000 53	−0.010 34	−0.000 89	−0.001 09

Further relevant data:
$e_1 = e_2 = 0$, $dw/dz = 0.001\,98$,
$t_{zz}^{(1)} = -32.929\,\text{N/mm}^2$, $t_{zz}^{(2)} = -13.872\,\text{N/mm}^2$, $t_{zz}^{(3)} = -0.008\,\text{N/mm}^2$,
$F_1 = 33.01\,\text{N}$, $F_2 = 33.06\,\text{N}$,
$N_1 = 95.67\,\text{N/mm}$, $N_2 = 87.27\,\text{N/mm}$.
$\Delta\varphi_1 = 0.075°$, $\Delta\varphi_2 = 0.086°$.

(The values of $E^{(1)}$, $\nu^{(1)}$, $E^{(3)}$ and $\nu^{(3)}$ appear to be of little account). In [3] and [4] we have found some estimates pertinent to $\nu^{(2)}$. Eventually in this paper the following data have been retained.

$$\begin{aligned} E^{(1)} &= E^{(2)} = E^{(3)} = 4\,\mathrm{N/mm^2}, \\ \nu^{(1)} &= \nu^{3} = 0.4997, \quad \nu^{(2)} = 0.4999. \end{aligned} \tag{3.3}$$

Finally, if the steel is assumed to be elastic, we shall use

$$E_1^s = E_2^s = 2.1 \times 10^5\,\mathrm{N/mm^2}. \tag{3.4}$$

For prescribed values of φ_1 and φ_2 we can solve the linear equations of the preceding section for the nine unknowns, among which N_1 and N_2. From these the values of F_1 and F_2 can be calculated through the use of (2.14). The numerical procedure has been applied repeatedly in an algorithm in which we varied the values of φ_1 and φ_2 in a neighbourhood of the critical pitch angle $\varphi_c \sim 35.26°$, until

$$|F_1 - F_2| < 10^{-1}\,\mathrm{N}, \tag{3.5}$$

after which the process was stopped. From the values φ_1, φ_2 obtained in this way for inextensible wires, we have singled out the pair

$$\varphi_1 = \bar{\varphi}_1 = 33.31° \quad \text{and} \quad \varphi_1 = \bar{\varphi}_2 = 37.08°, \tag{3.6^1}$$

which yields

$$F_1 = 33.01\,\mathrm{N} \quad \text{and} \quad F_2 = 33.06\,\mathrm{N}. \tag{3.6^2}$$

We note that $\bar{\varphi}_1 < \varphi_c$ and $\bar{\varphi}_2 > \varphi_c$. The values seem to conform to normal practice in manufacturing plants. Further we have found

$$\Delta\varphi_1 = 0.075° \quad \text{and} \quad \Delta\varphi_2 = 0.086°. \tag{3.7}$$

Apparently, φ_1 rotates towards φ_c, while φ_2 turns farther away from it. More details of this calculation can be found in Table 1.

If we repeat the same computation retaining the same combination of $\bar{\varphi}_1$ and $\bar{\varphi}_2$, but now for the case of elastic wires, we arrive at $F_1 = 33.24\,\mathrm{N}$ and $F_2 = 32.71\,\mathrm{N}$ (Table 2). Obviously, the effect of the finite steel elasticity is to be neglected as far as the wire forces are concerned. Moreover, we see that the stresses in the inner cylinder are about hydrostatic and that the outer cylinder does not matter. This was confirmed by the results of a calculation in which we omitted the inner and outer cylinders, applying the internal pressure q directly at the inner braid. This yielded the wire forces $F_1 = 32.93\,\mathrm{N}$ and $F_2 = 33.22\,\mathrm{N}$ (Table 3). For the sake of completeness we mention that for pairs φ_1, φ_2 in the immediate neighbourhood of φ_c, the presuppositions elastic or inextensible steel wires do mean large differences of the results.

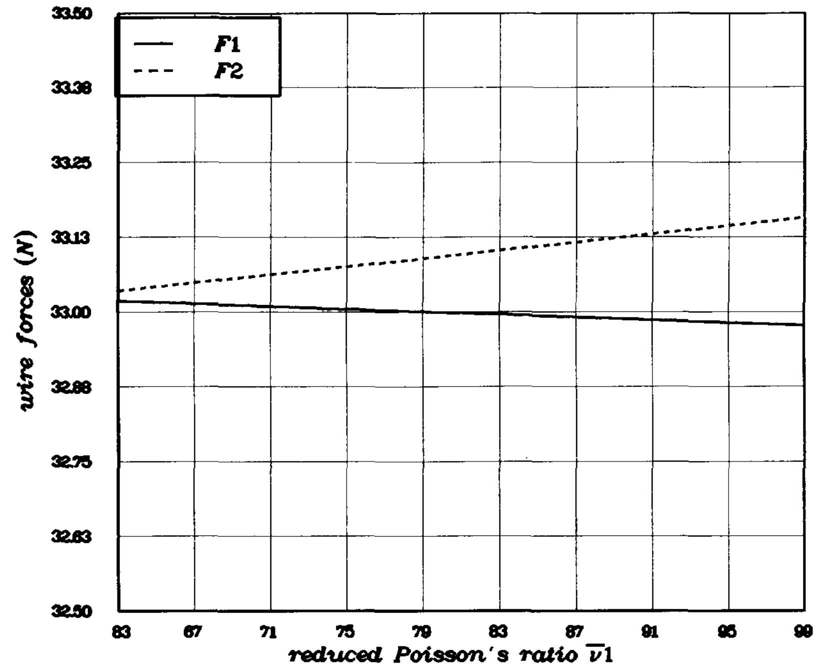

Fig. 3. The wire forces F_1 and F_2 as a function of the reduced Poisson's ratio $\bar{\nu}_1$.

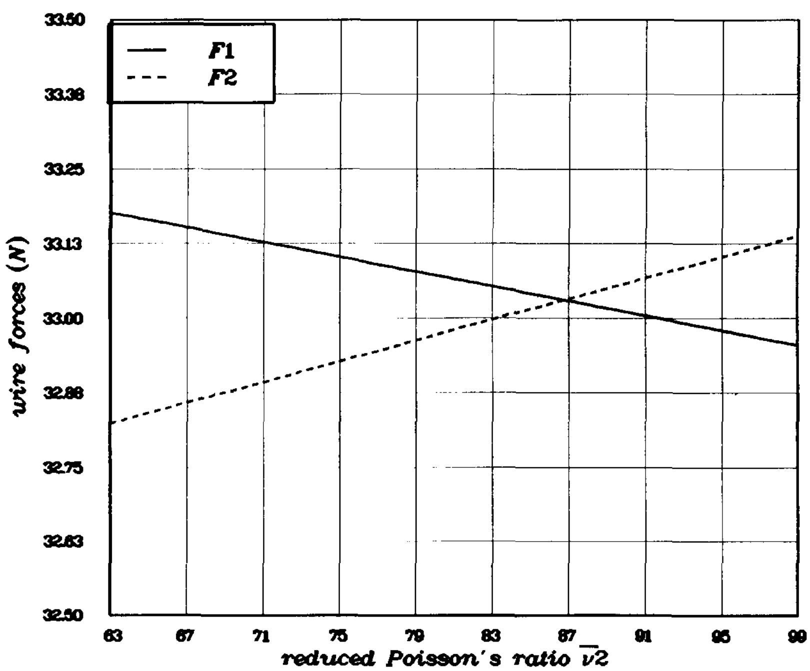

Fig. 4. The wire forces F_1 and F_2 as a function of the reduced Poisson's ratio $\bar{\nu}_2$.

Table 2. Trelleborg hose 320/10

Stresses, strains and displacements for the case:

$E^{(1)} = E^{(2)} = E^{(3)} = 4\,\mathrm{N/mm^2}$, $E_1^s = E_2^s = 210\,000\,\mathrm{N/mm^2}$, $d_1 = d_2 = 0.3\,\mathrm{mm}$,
$\nu^{(1)} = \nu^{(3)} = 0.4997$, $\nu^{(2)} = 0.4999$, $n_1 = 140$, $n_2 = 160$, $q = 33\,\mathrm{N/mm^2}$,
$\varphi_1 = 33.31°$, $\varphi_2 = 37.08°$.

i	r (mm)	$t_{rr}^{(i)}$ (N/mm²)	$t_{\vartheta\vartheta}^{(i)}$ (N/mm²)	$u^{(i)}$ (mm)	$e_{rr}^{(i)}$	$e_{\vartheta\vartheta}^{(i)}$
	4.750	−33.000 00	−32.8691	0.057 37	−0.037 01	0.012 08
1	5.875	−32.977 33	−32.891 7	0.021 01	−0.028 51	0.003 58
	7.000	−32.964 68	−32.904 4	−0.008 16	−0.023 77	−0.001 17
	7.000	−13.744 28	−13.718 0	−0.008 16	−0.011 02	−0.001 17
2	7.500	−13.742 50	−13.719 7	−0.013 50	−0.010 38	−0.001 80
	8.000	−13.741 20	−13.721 1	−0.018 56	−0.009 861	−0.002 32
	8.000	−0.002 12	0.012 48	−0.018 56	−0.007 80	−0.002 32
3	8.750	−0.000 93	0.011 28	−0.024 23	−0.007 35	−0.002 77
	9.500	0.000 00	0.010 34	−0.029 61	−0.007 00	−0.003 12

Further relevant data:
$e_1 = 0.002\,24$, $e_2 = 0.002\,20$, $\mathrm{d}w/\mathrm{d}z = 0.0101$,
$t_{zz}^{(1)} = -32.874\,\mathrm{N/mm^2}$, $t_{zz}^{(2)} = -13.687\,\mathrm{N/mm^2}$, $t_{zz}^{(3)} = -0.0457\,\mathrm{N/mm^2}$,
$F_1 = 33.24\,\mathrm{N}$, $F_2 = 33.71\,\mathrm{N}$,
$N_1 = 96.32\,\mathrm{N/mm}$, $N_2 = 86.34\,\mathrm{N/mm}$.
$\Delta\varphi_1 = 0.2968°$, $\Delta\varphi_2 = 0.3429°$.

Table 3. Trelleborg hose 320/10

Stresses, strains and displacements for the case:

$E^{(2)} = 4\,\mathrm{N/mm^2}$, $E_1^s = E_2^s = \infty$,
$\nu^{(2)} = 0.4999$, $n_1 = 140$, $n_2 = 160$, $q = 33\,\mathrm{N/mm^2}$,
$\varphi_1 = 33.31°$, $\varphi_2 = 37.08°$.

i	r (mm)	$t_{rr}^{(i)}$ (N/mm²)	$t_{\vartheta\vartheta}^{(i)}$ (N/mm²)	$u^{(i)}$ (mm)	$e_{rr}^{(i)}$	$e_{\vartheta\vartheta}^{(i)}$
	4.750	−33.000 00	−33.000 0	hydrostatic stresses assumed		
1	5.875	−33.000 00	−33.000 0			
	7.000	−33.000 00	−33.000 0			
	7.000	−13.955 70	−13.949 4	−0.006 00	−0.003 22	−0.000 858
2	7.500	−13.955 30	−13.9498	−0.007 58	−0.003 06	−0.000 938
	8.000	−13.955 00	−13.950 1	−0.009 08	−0.002 94	−0.001 135
	8.000	deleted				
3	8.750					
	9.500					

Further relevant data:
$e_1 = e_2 = 0$,
$t_{zz}^{(1)} = 33.\,\mathrm{N/mm^2}$, $t_{zz}^{(2)} = -13.941\,\mathrm{N/mm^2}$, $t_{zz}^{(3)} = -0\,\mathrm{N/mm^2}$,
$F_1 = 32.93\,\mathrm{N}$, $F_2 = 33.22\,\mathrm{N}$,
$N_1 = 95.43\,\mathrm{N/mm}$, $N_2 = 87.70\,\mathrm{N/mm}$.

Next, we have investigated the effect of the compressibility of the rubber used in the three cylinders for the case of inextensible steel, considered in Table 1. A variation of $\nu^{(3)}$ in the neighbourhood of the assumed value 0.4997 did not result in noticeable changes of the two braid forces. The effect of a modification of $\nu^{(1)}$ and $\nu^{(2)}$ is shown in Fig. 3 and Fig. 4, respectively. For the presentation we have used a reduced Poisson ratio $\bar{\nu}^{(i)}$,

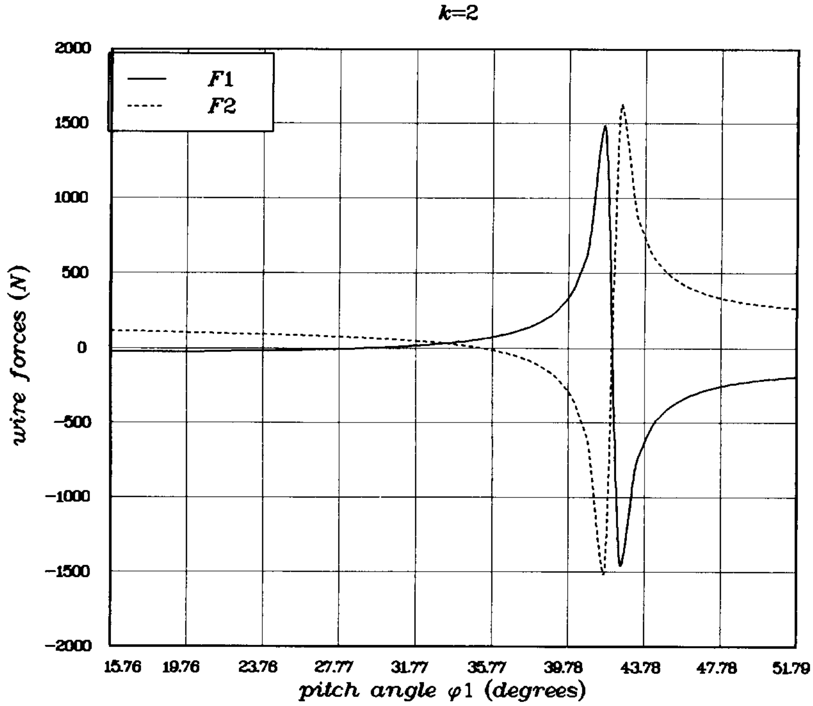

Fig. 5. The wire forces F_1 and F_2 along the line $\varphi_2 - \bar{\varphi}_2 = (\varphi_1 - \bar{\varphi}_1) \tan \frac{1}{8}\pi$.

defined by

$$\bar{\nu}^{(i)} = (\nu^{(i)} - 0.499\,00) \times 10^5, \quad i = 1, 2. \tag{3.8}$$

The effect of $\nu^{(2)}$ is large, that of $\nu^{(1)}$ small, as was to be expected.

Our main concern, however, is to see how the wire forces behave when φ_1 and φ_2 vary about a couple of prescribed values at which the two forces are much the same. This on account of the fact that in the manufacturing process it is impossible to avoid small deviations of the braid angles from specified values. In this relation we have investigated what happens in the neighbourhood of the pair $\bar{\varphi}_1$, $\bar{\varphi}_2$ (3.6)^1, through which we obtained the nearly equal forces (3.6)^2. We have varied φ_1 and φ_2 according to

$$\varphi_2 - \bar{\varphi}_2 = (\varphi_1 - \bar{\varphi}_1) \tan\left(\frac{k-1}{8}\pi\right), \quad k = 1, 2, \ldots, 8, \tag{3.9}$$

representing straight lines in the φ_1, φ_2-plane through the point $\bar{\varphi}_1$, $\bar{\varphi}_2$. Along each of the eight lines we have calculated F_1 and F_2. It appeared that the fluctuations of these quantities were much larger for $k = 2$ to 3, than for the remaining values of k. The smallest deviations occurred for $k \sim 7$. The results for $k = 2$, 4 and 6 are shown in Figures 5, 6 and 7, respectively. We note that invariably in the immediate neighbourhood of the pair $\bar{\varphi}_1$, $\bar{\varphi}_2$ there exists a second couple presenting coalescent F_1 and F_2 as well.

In view of these unexpected findings we have drawn Fig. 8, showing the difference $F_1 - F_2$ as a function of φ_1 and φ_2 about the critical value φ_c. We see again that the difference

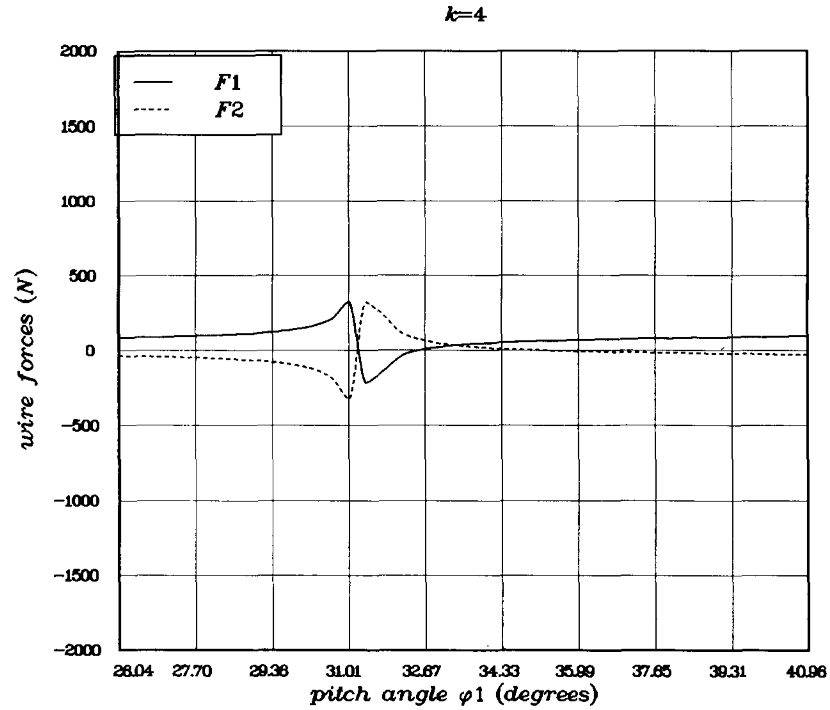

Fig. 6. The wire forces F_1 and F_2 along the line $\varphi_2 - \bar{\varphi}_2 = (\varphi_1 - \bar{\varphi}_1)\tan\frac{3}{8}\pi$.

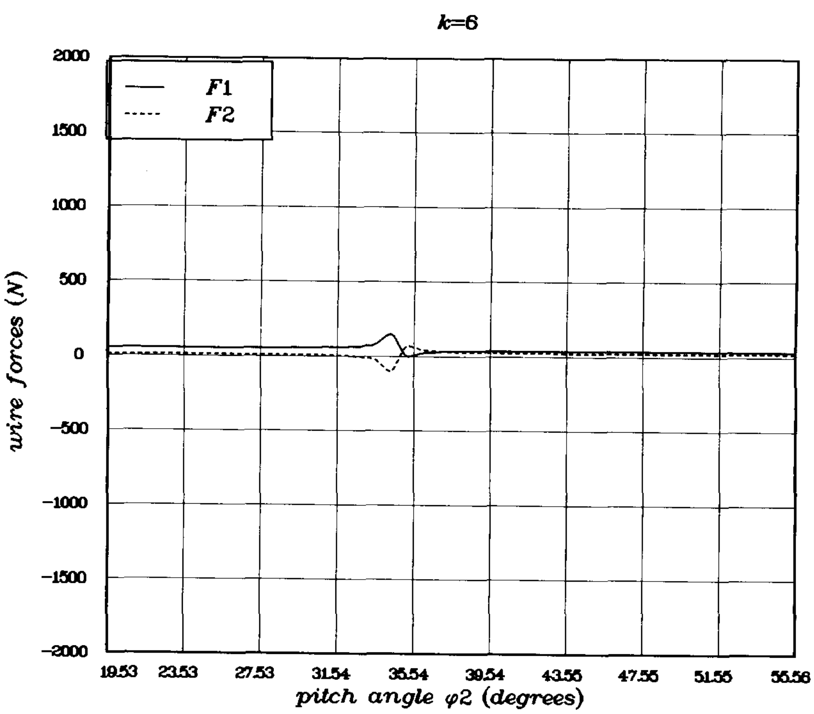

Fig. 7. The wire forces F_1 and F_2 along the line $\varphi_2 - \bar{\varphi}_2 = (\varphi_1 - \bar{\varphi}_1)\tan\frac{5}{8}\pi$.

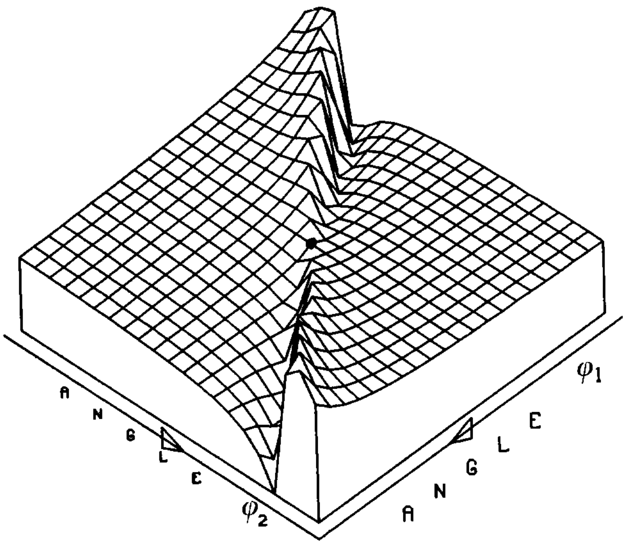

Fig. 8. The difference $F_1 - F_2$ as a function of φ_1 and φ_2 in the region $\varphi_c - 2.5° \leqslant \varphi_{1,2} \leqslant \varphi_c + 2.5°$. The dot denotes the point $\varphi_1 = \varphi_2 = \varphi_c$, where $F_1 - F_2 \cong 63$ N. The maximum value $F_1 - F_2 = 830$ N is attained at $\varphi_1 = 37.76°$ and $\varphi_2 = 37.50°$, and the minimum -760 N at $\varphi_1 = 37.76°$ and $\varphi_2 = 36.97°$.

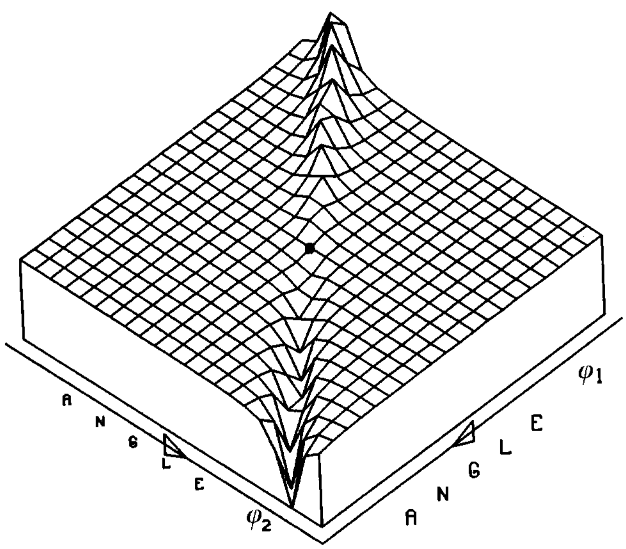

Fig. 9. The axial strain dw/dz as a function of φ_1 and φ_2 in the region $\varphi_c - 2.5° \leqslant \varphi_{1,2} \leqslant \varphi_c + 2.5°$. The dot denotes the point $\varphi_1 = \varphi_2 = \varphi_c$, where $dw/dz \cong 9 \times 10^{-6}$. The maximum strain $dw/dz = +0.90$ occurs at $\varphi_1 = 33.02°$ and $\varphi_2 = 33.55°$, and the minimal strain -1.14 at $\varphi_1 = 37.76°$ and $\varphi_2 = 37.23°$.

$F_1 - F_2$ varies little in the second and fourth quadrant of the φ_1, φ_2-plane ($\varphi_1 < \varphi_c$, $\varphi_2 > \varphi_c$ and $\varphi_1 > \varphi_c$, $\varphi_2 < \varphi_c$, respectively). On the contrary, in the first and third quadrant, i.e., for both angles larger or smaller than the critical value, large fluctuations of $F_1 - F_2$ occur. Hence, the pairs φ_1, φ_2 have to be taken from the second or fourth quadrant preferably.

Apart from the spreading of the load over the braids, the magnitude and the sign of the axial strain is a matter of some concern to designers of hoses as well. It is known that additional tensile forces acting at the hose accidentally, are carried mainly by the rubber. In order to avoid these extra tensile stresses one should try for a positive axial strain, preferably as small as possible. On the other hand, in the manufacturing process one often wishes this strain to be negative. Hence, we have plotted the axial strain as a function of φ_1 and φ_2 in Fig. 9, again in the neighbourhood of $\varphi_1 = \varphi_2 = \varphi_c$. Comparing this figure with Fig. 8, we note some similarity. Just like the force difference, the axial strain does not show large variations in the second and fourth quadrant, while exhibiting strong fluctuations in the remaining part of the φ_1, φ_2-plane. However, opposite to the behaviour of the force difference which shows positive as well as negative oscillations in the first and third quadrants, the large deviations of the axial strain do not change sign in these regions. We note that they are positive in the third quadrant, and negative in the first one.

4. Discussion

As mentioned before, the calculations in this paper rest on assumptions and numerical data, some of which are open to discussion. E.g., we call to mind the assumption of infinitesimally thin braids and the associated estimate of a "representative" value 0.4999 of the Poisson ratio ν_2 of the intermediate rubber layer. Therefore we have to consider the numerical results with some reservedness. However, we surmise that these shortcomings do not detract from the somewhat curious findings of Section 3, regarding the most desirable insertion of a pair φ_1, φ_2 in the neighbourhood of the critical angle φ_c in the φ_1, φ_2-plane.

We do not know yet what will happen when finite values of the displacements are accounted for in a geometrically non-linear theory. Most likely, for values of φ_1 and φ_2 near to φ_c, the finite increments $\Delta\varphi_1$ and $\Delta\varphi_2$ will give rise to a substantial departure from the results following from a linear theory. It seems worth-while to investigate this.

References

1. B.A. van den Horn and M. Kuipers, Strength and stiffness of a reinforced flexible hose, to appear in *Appl. Sci. Res.*
2. S.C. Hunter, *Mechanics of continuous media*, Ellis Horwood Limited, Chichester (1976).
3. P.C. Bregman, Analysis of rubber hose reinforced by a steel braid (in Dutch), Trelleborg B.V., Hoogezand (July 1986).
4. T. Nagy, F. Kovács, K. Lengyel and A. Edelényi, Improved mechanical model of reinforced hoses, Int. Rubber Conference, Stuttgart, Poster 6, (June 1985) 51–52.
5. H. Schilling, Gleichgewichtswinkel bei Verstärkungseinlagen in Gummischläuchen, *Kautschuk und Gummi-Kunststoffe* 25 (1972) 319–321.